

# Orbit determination for LISA beam acquisition

W. M. Folkner

Jet Propulsion Laboratory

California Institute of Technology

## Abstract

The LISA mission involves three spacecraft separated by 5 million kilometers. The three spacecraft need to be able to point at the other spacecraft in order to transmit and receive laser signals to each other. The initial beam acquisition process is dependent on the pointing accuracy of the spacecraft, the laser beam width, and the knowledge of the directions to the other spacecraft. The directions from one spacecraft to another will be determined by tracking radio signals from each spacecraft independently by Earth tracking stations. An analysis of the position determination estimation is described here. Using conventional Doppler and range information, the relative angular positions of each spacecraft can be determined to better than 1 microradian (0.2 arcseconds), which is less than the laser beamwidth.

## Introduction

This memo describes a covariance analysis for determining the relative positions of the three LISA spacecraft for the purpose of initial acquisition of laser beams. This analysis was motivated by the problem of initial signal acquisition. Previous studies focused on arm length uncertainty for reduction of the science measurements.

If the spacecraft pointing uncertainty is limited by a typical star track accuracy of order 1 arcsecond, then at a separation of 5 million km, the direction finding uncertainty translates into a position uncertainty of  $\sim 25$  km, which implies a need for only a modest orbit determination accuracy. Then, since the laser beam is  $\sim 0.5$  arcsecond, some type of raster search, or expansion of the beam, is necessary for initial beam acquisition. However, an improved star tracker accuracy may be achievable, perhaps by using the LISA inertial sensor to stabilize the angular motion of the star tracker and hence allow longer integration times. In that case, it may be desirable to have as precise an orbit determination accuracy as possible, to potentially eliminate the need for a search or beam expansion for signal acquisition.

For this memo, a nominal set of spacecraft trajectories was used. The nominal set is for assumed start of science operations near January 1, 2001. While the actual launch date will now be considerably later, the nominal orbit geometry should be almost identical and should not affect the estimated orbit determination accuracy. However, the orbit determination accuracy will depend on time of year over which tracking data acquired, due to the effect of the Earth's rotation on the tracking data in determining spacecraft position being dependent on spacecraft declination. This dependence was not explicitly studied. The orbit determination is expected to be worse for spacecraft declination near  $0^\circ$ . For the nominal orbits and tracking data used for this analysis, the declination of the spacecraft ranged between  $7^\circ$  and  $12^\circ$ , compared to a maximum values of  $\sim 23^\circ$  possible. Thus it would be expected in some case to have better orbit determination and in some cases poorer orbit determination.

## Spacecraft Trajectories

The initial states of the nominal trajectories are given in Table 1. These trajectories were numerically integrated with forces determined by the positions of the sun and planets as determined in the JPL planetary ephemeris DE200. No non-gravitational forces were used in the integration. Instead it was assumed that for these initial conditions, after separation (if necessary) of the transfer propulsion stage, the spacecraft would be placed into drag-free operation. This will produce more accurate orbit determination for purpose of estimating possible corrections to the

spacecraft orbits for science operation and for determining the pointing directions for laser signal acquisition.

### Tracking Data Schedule and Assumed Data Accuracy

Simulated tracking measurements were scheduled for the three spacecraft starting January 1, 2001 through January 25, 2001. The assumed tracking schedule was based on a 10-hour tracking pass of each spacecraft by a single antenna of the Deep Space Network on alternate days. Thus the total data accumulated for the orbit determination included 8 tracking passes of 10 hour duration for each spacecraft.

*Table 1. Spacecraft initial conditions, in Earth-centered Cartesian coordinates with respect to the Earth's mean equator of 2000, at epoch 01-JAN-2001 00:00:50.00 UTC*

	Spacecraft 1	Spacecraft 2	Spacecraft 3
X (km)	4.9864966754E+07	5.3053829265E+07	5.4737801861E+07
Y (km)	3.0535278380E+06	2.1217600466E+06	2.4370626194E+06
Z (km)	2.0900637631E+06	-1.6854308267E+06	3.0153038721E+06
Vx (km/s)	4.4318500886E-01	1.4730133383E-01	6.4631459980E-01
Vy (km/s)	9.3435539352E+00	9.9630069694E+00	1.0090625999E+01
Vz (km/s)	4.5696138104E+00	4.1810176386E+00	3.9927403004E+00

For each tracking pass, simulated range and range-rate (Doppler) data points were simulated at 60 second intervals. For these data, the assumed accuracy was assumed to be a random 1 m error in range and 0.1 mm/s for range-rate. These data rates and accuracies are considered typical for DSN interplanetary spacecraft tracking data, for spacecraft a Sun-Earth-spacecraft angles of greater than 45°. In fact, because the LISA spacecraft will be closer than typical interplanetary spacecraft (0.3 AU) and at 90° Sun-Earth-spacecraft angle, the actual data accuracy is likely to be better than this, by a factor of perhaps 2-3. Thus the orbit determination accuracy given here may be a bit conservative and might be revised downwards in future studies.

### Modeling Assumptions

The spacecraft parameters and error models used in the orbit determination analysis are listed in Table 2. The spacecraft initial conditions are estimated with a large a priori uncertainty, since the final estimated uncertainty is expected to be much improved after the orbit solution and the initial conditions will be significantly perturbed by separation from the transfer propulsion stage. The DSN station locations are currently known to 3 cm accuracy for the antennas used most often. The media delay uncertainties are typical of the DSN sites after calibration (using a seasonal model for the troposphere and dual-band GPS data for the ionosphere). The media uncertainties are treated as uncorrelated from one tracking pass to the next. The uncertainty in the knowledge of the orbits of the planets, including the Earth, about the sun are usually an important error source for interplanetary spacecraft orbit determination. However, for LISA the orbit determination requirements are primarily for one spacecraft respect to another. Since these are all based on Earth tracking data, the uncertainty of the planetary orbits is common to all spacecraft and cancels out in the differences (and for position with respect to the Earth).

Table 2. Orbit determination parameters and a priori uncertainties

Parameter	A priori uncertainty (1 $\sigma$ )
Spacecraft initial position	100 km (each of three components)
Spacecraft initial velocity	1 m/sec (each of three components)
Tracking station location	3 cm (each of three components)
Earth orientation	5 nrad per component
Zenith wet troposphere path delay	4 cm (daily)
Zenith dry troposphere path delay	1 cm (daily)
Zenith ionosphere path delay	$5 \times 10^{16}$ electrons/cm <sup>2</sup> (daily)

### Orbit Determination Results

Table 3 gives the covariance matrix for the positions of the three LISA spacecraft at the end of the 24 day tracking arc. This covariance matrix is expressed in Earth-mean-equator of 2000 Cartesian coordinates. In order to determine the uncertainty in pointing direction from one spacecraft to another, the covariance matrix for each pair of spacecraft is mapped to a coordinate system where one axis ( $r_-$ ) is in the direction between the two spacecraft and the other two coordinates ( $t_-$  and  $p_-$ ) are in (arbitrary) transverse directions. In order to determine the appropriate coordinate systems, the spacecraft positions at the end of the tracking arc, given in Table 4, are used. The coordinate systems used for the transformation of the covariance of each spacecraft pair are given in Table 5. The 6x6 position covariance matrix for each spacecraft pair was transformed by the relation

$$s^2_{ij} = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} s^2_{kl} \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix}^T$$

where  $s^2$  is the 6x6 covariance matrix and  $R$  is the 3x3 matrix describing the ( $r_-$ ,  $t_-$ ,  $p_-$ ) unit vectors in terms of the ( $x$ ,  $y$ ,  $z$ ) unit vectors. For the transformed covariance, the uncertainty in the relative position in each coordinate direction is given by  $s^2_{aa} = s^2_{aa,i} + s^2_{aa,j} - 2s^2_{aa,ij}$ , where  $a$  is one of ( $r_-$ ,  $t_-$ ,  $p_-$ )  $s^2_{aa,i}$  is the covariance element in the  $a$  component for spacecraft  $i$  and  $s^2_{aa,ij}$  is the cross-covariance element correlating spacecraft  $i$  and  $j$ . The resulting uncertainties in the three directions for each spacecraft pair are given in Table 6.

Table 3. Spacecraft position covariance matrix at end of tracking arc (km<sup>2</sup>)

	x LISA1	y LISA1	z LISA1	x LISA2	y LISA2	z LISA2	x LISA3	y LISA3	z LISA3
x LISA1	1.98E-2	1.31E-1	-3.35E-1	-1.44E-3	1.48E-2	-3.70E-2	6.70E-4	1.35E-2	-3.12E-2
y LISA1	1.31E-1	1.90E+0	-4.23E+0	-2.04E-2	2.07E-1	-5.16E-1	8.53E-3	1.94E-1	-4.44E-1
z LISA1	-3.35E-1	-4.23E+0	9.60E+0	4.57E-2	-4.64E-1	1.16E+0	-1.94E-2	-4.33E-1	9.95E-1
x LISA2	-1.44E-3	-2.04E-2	4.57E-2	2.82E-2	-2.18E-1	4.96E-1	-8.24E-4	-2.22E-2	5.03E-2
y LISA2	1.48E-2	2.07E-1	-4.64E-1	-2.18E-1	2.01E+0	-4.86E+0	8.39E-3	2.21E-1	-5.02E-1
z LISA2	-3.70E-2	-5.16E-1	1.16E+0	4.96E-1	-4.86E+0	1.20E+1	-2.10E-2	-5.51E-1	1.25E+0
x LISA3	6.70E-4	8.53E-3	-1.94E-2	-8.24E-4	8.39E-3	-2.10E-2	1.18E-2	4.50E-2	-1.48E-1
y LISA3	1.35E-2	1.94E-1	-4.33E-1	-2.22E-2	2.21E-1	-5.51E-1	4.50E-2	1.75E+0	-3.85E+0
z LISA3	-3.12E-2	-4.44E-1	9.95E-1	5.03E-2	-5.02E-1	1.25E+0	-1.48E-1	-3.85E+0	8.72E+0

*Table 4. Spacecraft initial conditions, in Earth-centered Cartesian coordinates with respect to the Earth's mean equator of 2000, at epoch 25-JAN-2001 00:00:50.00 UTC*

	Spacecraft 1	Spacecraft 2	Spacecraft 3
X (km)	4.6359658352E+07	4.8672827340E+07	5.1246079753E+07
Y (km)	2.2270389751E+07	2.2200026363E+07	2.3277265819E+07
Z (km)	1.1404665276E+07	6.9673349687E+06	1.1119632128E+07
Vx (km/s)	-3.8119806000E+00	-4.3117444000E+00	-4.0027504000E+00
Vy (km/s)	8.9455531000E+00	9.1261558000E+00	9.7048830000E+00
Vz (km/s)	4.2929404000E+00	4.0437446000E+00	3.7022942000E+00

*Table 5. Pointing coordinate system for the spacecraft pairs*

	Spacecraft 1-2			Spacecraft 2-3			Spacecraft 1-3		
	x	y	z	x	y	z	x	y	z
r_	-0.462	0.014	0.887	-0.514	-0.215	-0.830	-0.978	-0.201	0.057
t_	0.279	0.951	0.130	-0.749	0.584	0.313	-0.077	0.600	0.796
p_	-0.842	0.307	-0.444	0.418	0.782	-0.462	-0.195	0.774	-0.602

*Table 6. Relative spacecraft position uncertainties (1-sigma)*

	Spacecraft 1-2	Spacecraft 2-3	Spacecraft 1-3
r_ uncertainty (km)	3.9	3.2	0.7
t_ uncertainty (km)	1.2	0.4	2.2
p_ uncertainty (km)	2.6	3.4	3.8

The uncertainties in the r\_ direction correspond to uncertainties in the distance between spacecraft. The values given in Table 6 are comparable to previous results, such as those given in the LISA Pre-Phase A Report, Second Edition, in Table 6.2. The difference can be explained by differences in the amount of tracking data and by slightly different modeling of the Earth media errors. The uncertainties in the transverse directions divided by the distance between spacecraft, 5 million km, gives the a priori uncertainty in pointing angle. The 1-sigma uncertainty is approximately bounded by  $4 \text{ km}/5 \times 10^6 \text{ km}$  which is 0.8 microradian or  $\sim 0.15$  arcsecond.

## Conclusion

With a laser beam diameter of  $\sim 10^{-6} \text{ m}/0.3 \text{ m} = 3$  microradian, the 3-sigma uncertainty in a priori pointing direction is approximately equal to the beamwidth. This might make it marginally possible to directly acquire the laser beams rather than having to engage in a raster search or diverge the beams. If this is a possible reduction in mission cost and complexity it should be considered. It should be kept in mind that, though the 3-sigma pointing uncertainties are close to the required values, smaller orbit determination position uncertainties can almost certainly be achieved. A significant improvement might be achieved by a more accurate assessment of the tracking data noise for the actual LISA geometry. If insufficient margin is achieved, it is definitely possible to achieve smaller orbit determination accuracy through differential VLBI tracking of the spacecraft. For VLBI measurement, tracking stations at two DSN complexes would simultaneously track each spacecraft in turn, using widely spaced tones about the carrier, to achieve an instantaneous relative plane-of-sky position measurement. This is a proven technique, with accuracy of  $\sim 5$  nanoradian or  $< 1 \text{ km}$  in position uncertainty at the distance of the LISA spacecraft. VLBI tracking is a bit more complex operationally and requires tones on the spacecraft radio system, but potentially saves a significant amount of antenna time and reduces the time for the orbit solution.